

LTL Model Composition

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– handout –

Motivation

- ▶ ω -word, e.g. $\alpha = \binom{1}{0} \binom{0}{0} \binom{1}{1} \binom{1}{1} \binom{0}{1} \binom{1}{0} \binom{0}{1} \binom{1}{0} \dots$
- ▶ LTL formula, e.g. $\gamma = FG(p \leftrightarrow Xq)$

(LTL-)Model-Checking: Does α satisfy γ ?

Classical solution: Büchi automata

exponential

Alternative approach: Composition technique
(Shelah – for MSO logic and orderings)

non-elementary

- ▶ for disjoint sum of components, e.g. concatenation of words

$$\alpha = \underbrace{\binom{1}{0} \binom{0}{0} \binom{1}{1} \binom{1}{1}}_{w_0} \underbrace{\binom{0}{1} \binom{1}{0} \binom{1}{1}}_{w_1} \underbrace{\binom{0}{1} \binom{1}{0}}_{w_2} \dots$$

- ▶ reduce MC problem for sum to MC problems for components

Here: Composition for LTL (in NNF) on ω -words

exponential

Preparation of Theorem

| | | | | | | |
|---------|------------|-------|-------|-------|-------|---------|
| models | $\alpha =$ | w_0 | w_1 | w_2 | w_3 | \dots |
| indices | | 0 | 1 | 2 | 3 | \dots |

- ▶ Given LTL formula γ
- ▶ Goal: fix enough information to decide whether $\alpha \models \gamma$
- ▶ Introduce formulas $\gamma_1, \dots, \gamma_m$ interpreted in the models w_0, w_1, w_2, \dots

- ▶ This introduces sets X_1, X_2, \dots, X_m :

$$X_k = \{i \mid w_i \models \gamma_k\}$$

X_k tells in which model w_i the formula γ_k is true

- ▶ Introduce MSO formula $\beta(X_1, \dots, X_m)$ interpreted in $(\mathbb{N}, <)$

Theorem

Given: signature $\sigma = \{p_1, \dots, p_r\}$, fixed index ordering $(\mathbb{N}, <)$

For every σ -LTL formula γ in NNF we can compute

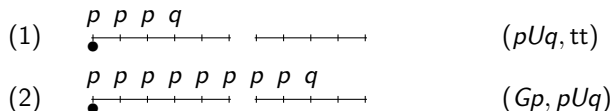
- ▶ σ -LTL formulas $\gamma_1, \dots, \gamma_m$
- ▶ MSO formula $\beta(X_1, \dots, X_m)$ interpreted in index ordering

such that for every word α :

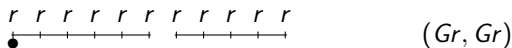
$$\alpha \models \gamma \Leftrightarrow \beta(X_1, \dots, X_m) \text{ holds in index ordering}$$

Number of formulas $\gamma_1, \dots, \gamma_m$ is exponential in the number of all subformulas of γ – denoted by $cl(\gamma)$.

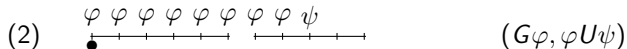
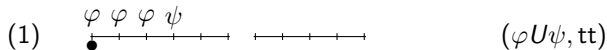
Idea of Composition for pUq :



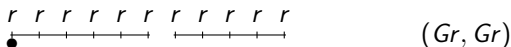
Idea of Composition for Gr :



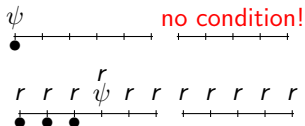
Idea of Composition for $\varphi U \psi$?



Idea of Composition for Gr :

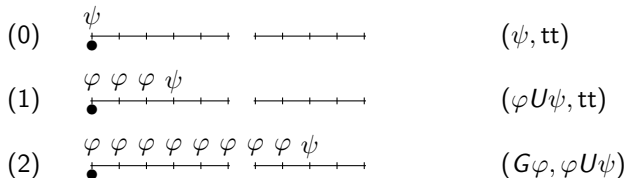


$\varphi U \psi$: Does this idea suffice? No! Consider (1) for $\varphi = Gr$

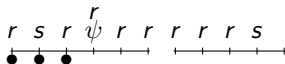


\Rightarrow Distinguish between ψ and $\varphi U \psi$ and $G\varphi$ in first component

Idea of Composition for $\varphi U \psi$?



Does this suffice? No! Consider again (1) for $\varphi = rUs$



For some states rUs may hold and for some $Gr!$

\Rightarrow Distinguish between $\underbrace{\psi, \varphi U \psi, G\varphi, (\varphi U \psi) \vee G\varphi}_{\text{cf}(\delta)}$ in 1. component

$\delta = \varphi U \psi$ - arbitrary number of components

- ▶ Formulas for first component:

$$\text{cf}(\delta) = \{\bar{\psi}, \bar{\varphi} U \bar{\psi}, G\bar{\varphi}, (\bar{\varphi} U \bar{\psi}) \vee G\bar{\varphi}' \mid \bar{\varphi}, \bar{\varphi}' \in \text{cf}(\varphi), \bar{\psi} \in \text{cf}(\psi)\}$$

| $\bar{\delta}$ | at one state | at more states | $F(\bar{\delta})$ |
|--|--------------|----------------|--|
| $\bar{\psi}$ | | | $F(\bar{\psi})$ |
| $(\bar{\varphi} U \bar{\psi})$ | | | $F(\bar{\varphi}), F(\bar{\psi})$ |
| $G\bar{\varphi}$ | | | $\text{cf}(\delta), F(\bar{\varphi})$ |
| $(\bar{\varphi} U \bar{\psi}) \vee G\bar{\varphi}$ | | | $\text{cf}(\delta), F(\bar{\varphi}), F(\bar{\psi})$ |

- ▶ Formulas for next component: $F(\bar{\delta})$

inductively add $\text{cf}(\delta)$ for cases $\bar{\delta} = G\bar{\varphi} / \bar{\delta} = (\bar{\varphi} U \bar{\psi}) \vee G\bar{\varphi}$ at current component

Hints on formal representation

Description by interface information: $\langle \underbrace{\delta_1, \dots, \delta_k}_{F(\delta)}; \underbrace{\beta(X_1, \dots, X_k)}_{\text{MSO formula}} \rangle$

- ▶ β speaks about component indices
- ▶ X_j “contains all components” in which δ_j holds

β describes that for subsets of the X_j holds:

- ▶ an initial condition
“one of the formulas from $F(\delta)$ holds at first component”
- ▶ a local compatibility condition
“update rule from last slide holds for all components $i, i + 1$ ”
- ▶ a global compatibility condition
“for every $\bar{\varphi} U \bar{\psi}$: $G\bar{\varphi}$ does not hold on all components”

- ▶ Construction of interface information:

- ▶ $cf(\delta) = \{\bar{\psi}, \bar{\varphi}U\bar{\psi}, G\bar{\varphi}, (\bar{\varphi}U\bar{\psi}) \vee G\bar{\varphi}' \mid \bar{\varphi}, \bar{\varphi}' \in cf(\varphi), \bar{\psi} \in cf(\psi)\}$

- ▶ other cases similar

- ⇒ size of $cf(\gamma)$ is exponential in size of $cl(\gamma)$

- ▶ Initial conversion of formula in NNF:

- exponential blowup of size of $cl(\gamma)$

- ⇒ in total double exponential

Ideas for other formula types

- ▶ $X\varphi$: distinguish between
 - ▶ last state (“last”) $\Rightarrow \text{cf}(\varphi)$ at next component
 - ▶ $\neg\text{last} \wedge X\bar{\varphi}$
 - ▶ $\neg\text{last} \rightarrow X\bar{\varphi} \Rightarrow \text{cf}(\varphi)$ at next component
- ▶ $\varphi \vee \psi$: distinguish between
 - ▶ all states $\bar{\varphi}$
 - ▶ all states $\bar{\psi}$
 - ▶ some states $\bar{\varphi}$, some $\bar{\psi}$
- ▶ $\varphi R \psi$: analogous to $\varphi U \psi$
- ▶ $\varphi \wedge \psi$: simply all states $\bar{\varphi} \wedge \bar{\psi}$

Summary and Outlook

Summary:

- ▶ Composition technique for ordered sums and LTL logic
- ▶ Reduce MC problem for sum to MC problems for components
- ▶ Double exponential (instead of non-elementary)

Outlook (Ongoing work):

- ▶ Improve result: use FO logic instead of “simple” MSO
- ▶ Generalize result:
 - ▶ (All paths of) ordered sum of transition systems and LTL
 - ▶ Computational tree logic (CTL) instead of LTL
⇒ compose tree models